

CO6 “Introduction to Computational Neuroscience”

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course website: <https://lnc2.dec.ens.fr/en/teams/mathematics-neural-circuits/co6-course>

Exercise Sheet 3 — 2 April 2019

Please submit your solution in the next class (9 April 2019)

(1) **Signal Detection Theory:** Consider the two-alternative-choice motion discrimination task in which the subject has to determine the direction of motion of a stimulus. As ideal observers, we obtain access to two neurons with opposite tuning (so that e.g. neuron 1 will fire more strongly than neuron 2 if the motion stimulus moved leftwards, and neuron 2 more strongly than neuron 1 if the motion stimulus moved rightwards). If the motion was to the left (+), then the firing rates of neuron 1 follow the distribution $p(r|+)$ and those of neuron 2 the distribution $p(r|-)$; if the motion was to the right (-), neuron 1 fires according to $p(r|-)$ and neuron 2 according to $p(r|+)$.

(a) Assume that the distributions $p(r|+)$ and $p(r|-)$ are Gaussian and sketch the above scenario (two neurons, two possible stimuli).

(b) In class, we introduced the two functions

$$\alpha(z) = p(r \geq z|-) \tag{1}$$

$$\beta(z) = p(r \geq z|+) \tag{2}$$

Sketch these functions! What do they indicate in the present context?

(c) Compute the derivatives $d\alpha(z)/dz$ and $d\beta(z)/dz$. Hint: Remember how integral and derivative are related!

(d) Let us call the first neuron’s firing rate r_1 and the second neuron’s firing rate r_2 . What is the probability $p(r_1, r_2|+)$, i.e., the probability of observing the values r_1 and r_2 for the first and second neuron if the motion was leftwards (+)? Hint: Remember about probabilities: they add up to 1 for all the possible choices; probabilities of independent choices add up.

(e) **Advanced:** Let us assume an ideal observer makes the decision “motion was leftwards” whenever $r_1 > r_2$. How often will this observer be correct, assuming that leftwards and rightwards motion occurs equally often? (Hint: you need to integrate $p(r_1, r_2|+)$ over the correct area in the (r_1, r_2) -plane.) Show that the result corresponds to the area under the ROC-curve.

(2) **Reinforcement Learning Theory:** In the lecture, we learned that the reinforcement learning framework allows an agent to learn the optimal sequence of choices in a given environment. (Optimal in the sense that the agent will gather the maximum amount of reward.) This framework is very powerful and could in principle learn many things, e.g., playing chess, even if this may often be impractical. What are environmental scenarios which this framework, as presented in the lecture, will *never* be able to learn? If you can think of a scenario, try to illustrate it in a simple example.